Second Winner	Rank	◄
Basic Sciences	Category	◄
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Bosnian	Nationality	◄
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Explicit formulas in number theory	Project Title	◄

Abstract ◄
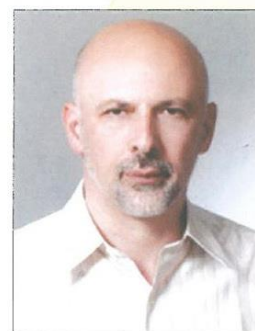
After A. Weil, the explicit formulas in number theory constitute a family of identities relating the values of a test function over prime powers to the values of its Mellin transform over the zeros of the corresponding zeta function. The role of primes can be taken by prime ideals of a number field, primitive hyperbolic classes of a Fuchsian group, etc. Continuing our investigation of the Jorgenson-Lang fundamental class of functions that encompasses a wide range of zeta and L-functions in number theory, we concentrate here on the problem of spectra of the Laplace-Beltrami operator of a non-compact Riemann surface with a finite volume. Approaching the Selberg trace formula as an explicit formula, we expect to obtain new results about the distribution of both the discrete and continuous spectra of this operator and about special values of the Selberg zeta function. The Selberg trace formula is presently the only available tool for addressing this open problem that is of relevance in many research areas, including quantum systems which do not permit exact solutions. New upper and lower bounds for the constant term of the logarithmic derivative of the Selberg zeta function are of interest in Arakelov geometry.

Realizing our research program, so far we have:

-0   enlarged the class of test functions to which Jorgenson-Lang explicit formula applies, by weakening the condition at infinity and the condition at zero;

-1   proved a new explicit formula valid in the setting not previously covered, when the fudge factor in the functional equation possesses infinitely many zeros and poles in the half of the critical strip

-2   further weakened the condition at zero in the symmetric case

-3   proved that the Selberg trace formula in the regularized sense holds for a larger class of test functions and applied a test function not satisfying Selberg's conditions in order to obtain a new representation of the logarithmic derivative of the Selberg zeta function, both in the compact and non-compact (finite volume) case

-4   improved the conditional resp. unconditional estimates of the growth of the logarithmic derivative of the Riemann zeta resp. Selberg zeta function and deduced the best known estimates of the error term in the prime geodesic theorem for cofinite Fuchsian groups.

-5   proved that the condition at zero might be completely omitted in the explicit formula for the hyperbolic scattering determinant and generalized this result to the case when

-6   derived Stieltjes type expressions for the higher order Euler constants on number fields, using a broader class of test functions in the explicit formula for the Dedekind zeta function $Z = 1/Z$

The applicability of the above theory spreads from number theory into the areas of representation theory, spectral theory and the theory of regularized determinants familiar in mathematical physics.